MARKING GUIDE FOR SEMINAR MATHEMATICS QUESTIONS 2022

1. (a) $\log_x 9+\log_{x^2} 3=2.5 \Rightarrow \log_{x^2} 3=\frac{1}{2}\log_x 3=\log_x \sqrt{3}$

$\log_x 9+\log_x \sqrt{2}=2.5$

$\log_x \sqrt[9]{.3}=2.5 \Rightarrow 9\sqrt{3}=x^{2.5}$

$\log_{10} \sqrt[9]{3}=2.5\log_{10}^{x} \Rightarrow \log_{10}^{3^{2.5}}=2.5\log_{10}^{x}$

$2.5\log_{10}^{3}=2.5\log_{10}^{x}$

$\Rightarrow x=3$

(b) $\log_5^{21}=m,\quad \log_9^{75}=n$

$21=5^m$

$7\times 2=5^m$

$\Rightarrow 75=9^n$

$\Rightarrow 25\times 3=3^{2n}$

$\Rightarrow 25=3^{2n-1}$

$\Rightarrow 5^2=3^{2n-1}$

$\Leftarrow 5=3^{\frac{2n-1}{2}}$

$3=5^{\frac{2}{2n-1}}$

$7\times 5^{\frac{2}{2n-1}}=5^m \Rightarrow 7=5^{m-\frac{2}{2n-1}}$

$5^{\frac{m(2n-1)-2}{2n-1}}$

$$\Rightarrow \log_5^7=\frac{m(2n-1)-2}{2n-1}=\frac{1}{2n-1}(2mn-m-2)$$

(c) $p^3+q^3=4$, $pq=\frac{1}{2}(p^3+q^3)+1=\frac{1}{2}(4)+1=3$

New roots, $\frac{1}{p^6}$ and $\frac{1}{q^6}$.

Sum of roots $\frac{1}{p^6}+\frac{1}{q^6}=\frac{p^6+q^6}{(pq)^6}=\frac{(p^3+q^3)2-2(pq)^3}{(pq)^6}$

$=\frac{(4)^2-2(3)^3}{(3)^6}$

¿$\frac{-38}{729}$

Product of roots $\frac{1}{p^6}.\frac{1}{q^6}=\frac{1}{(pq)^6}=\frac{1}{3^6}=\frac{1}{729}$.

Equation $X^2+\frac{38}{729}x+\frac{1}{729}=0\Rightarrow 729x^2+38+1=0$

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2. (a) $a=7,\quad a+(n-1)d=70\Rightarrow 7+(n-1)d=70\Rightarrow(n+1)d=63.$

$sn=\frac{n}{2}(2a+(n-1)d)=385\Rightarrow\frac{n}{2}(14+63)=385$

$77n=770$

$n=10$

$(n-1)d=63\Rightarrow 9d=63,\quad d=7$

(b) Let the first term be a and the common ration $r=0.9$

$a+0.9a+(0.9)^2a+(0.9)^3a=20$

$a(1+0.9+0.9^2+0.9^3)=20$

$2.439a=20\Rightarrow a=5.82$

He will walk $S\infty=\frac{a}{1-r}=\frac{5.82}{1-0.9}=58.2m$

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3. (a) (i) $\frac{1}{x^4-9}=\frac{1}{(x^2-3)(x^2+3)}=\frac{1}{(x-\sqrt{3})(x+\sqrt{3})(x^2+3)}$

$\Rightarrow\frac{1}{x^4-9}=\frac{A}{x-\sqrt{3}}+\frac{B}{x+\sqrt{3}}+\frac{cx+D}{x^2+3}$

$A(x+\sqrt{3})(x^2+3)+B(x-\sqrt{3})(x^2+3)+(cx+D)(x-\sqrt{3})(x+\sqrt{3})=1$

Put $x=\sqrt{3}$, $12\sqrt{3}A=1\Rightarrow A=\frac{1}{12\sqrt{3}}$

Put $x=0$, $3\sqrt{3}.\frac{1}{12\sqrt{3}}+\frac{3\sqrt{3}}{12\sqrt{3}}-3D=1\Rightarrow D=\frac{-1}{6}$

Put $x=1$, $\left(\frac{1+\sqrt{3}}{4}\right)-\left(\frac{1-\sqrt{3}}{4}\right)-2c+\frac{1}{3}=1\Rightarrow c=0$

$$\Rightarrow \frac{1}{x^4-9} = \frac{1}{12\sqrt{3}\left(x-\sqrt{3}\right)} + \frac{-1}{12\sqrt{3}\left(x+\sqrt{3}\right)} - \frac{1}{6\left(x^2+3\right)}$$

$$\Rightarrow \frac{\sqrt{3}}{36\left(x-\sqrt{3}\right)} - \frac{\sqrt{3}}{36\left(x+\sqrt{3}\right)} - \frac{1}{6\left(x^2+3\right)}$$

(ii) $$\frac{x^3-x^2-3x+5}{(x-1)\left(x^2-1\right)} = \frac{\left(x^3-x^2-x+1\right)-2x+4}{x^3-x^2-x+1}$$

$$=1+\frac{4-2x}{(x-1)\left(x^2-1\right)}$$

Consider $$\frac{4-2x}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$$

$$4-2x = A(x-1)^2 + B(x-1) + C(x+1)$$

Put $x=1$ $\quad 2C=2 \Rightarrow C=1$

$x=-1$ $\quad 4A=6 \Rightarrow A=3/2$

$x=0$ $\quad -B=3/2 \Rightarrow B=-3/2$

$$\therefore \frac{x^3-x^2-3x+5}{(x-1)\left(x^2-1\right)} = 1+\frac{3}{2(x+1)} - \frac{3}{2(x-1)} + \frac{1}{(x-1)^2}$$

(b) $n_{cr} = a^{n-r} b^r$

$$\left(2y-\frac{1}{y^2}\right)^{12} \qquad (2y)^{n-r}.\left(\frac{1}{y^2}\right)r = y^0$$

$$y^{12-r}.y^{-2r} = y^0$$

$\Rightarrow 12-3r=0 \qquad \Rightarrow r=4$

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4. (a) $$\frac{(2-i)^2(3i-1)}{i+3}$$

Vet $$z=\frac{(2-i)^2(-1+3i)}{3+i} = \frac{(5-4i)(-1+3i)}{3+i}$$

$$\Rightarrow \frac{7+19i}{3+i} \times \frac{(3-i)}{(3-i)} = \frac{21-7i+57i+19}{10} = \frac{40+50i}{10} = 4+5i$$

$$z = 4+5i, r = |z| = \sqrt{16+25} = \sqrt{41}$$

$$\theta = Arg(z) = \tan^{-1}\left(\frac{5}{4}\right) = 57.3^0$$

$$z = r(\cos\theta + i\sin\theta) = \sqrt{41}(\cos 51.3^0 + i\sin 51.3^0)$$

(b)

$$\frac{z-6i}{2z-1} = \frac{x+(y-6)i}{2x-1+2yi} = \frac{[x+(y-6)i]}{[(2x-1)+2yi]}\frac{[(2x-1)-2yi]}{[(2x-1)-2yi]}$$

$$= \frac{x(2x-1)-xyi+(2x-1)(y-6)i+2y(y-6)}{(2x-1)+y^2}$$

If it’s purely imaginary, then the real part = 0

$$\Rightarrow x(2x-1)+2y(y-6)=0$$

$$2x^2-2x+2y^2-12y=0 \Rightarrow x^2+y^2-x-6y=0$$

Comparing with $x^2+y^2+2gx+2fy+c=0$

$g = \frac{-1}{2}$, $f=3,$ $c=0$, centre $\left(\frac{1}{2}, 3\right)$ and radius = $\sqrt{\frac{1}{4}-\frac{1}{3}} = \sqrt{\frac{37}{2}}$

(c)

$$Arg\left(\frac{z-3}{z-2i}\right) = \frac{\pi}{4} \Rightarrow Arg(z-3) - Arg(z-2i) = \frac{\pi}{4}$$

$$Arg[(x-3)+iy] - Arg(x+(y-2)i) = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{y}{x-3}\right) - \tan^{-1}\left(\frac{y-2}{x}\right) = \frac{\pi}{3}$$

Let $\tan^{-1}\left(\frac{y}{x-3}\right) = A \tan A = \frac{y}{x-3}$

$$\tan^{-1}\left(\frac{y-2}{x}\right) = B \Rightarrow \tan B = \frac{y-2}{x}$$

$$\Rightarrow A-B=\frac{\pi}{4}, \qquad \tan(A-B)=1\Rightarrow\frac{\tan A-\tan B}{1-\tan A\tan B}=1$$

$$\tan A-\tan B=1-\tan A\tan B$$

$$\frac{y}{x-3}-\frac{y-2}{x}=1+\frac{y(y-2)}{x(x-3)}\Rightarrow\frac{yx-(y-2)(x-3)}{x(x-3)}$$

$$=\frac{x(x-3)+y(y-2)}{x(x-3)}$$

$$yx-yx+3y+2x-6=x^2-3x+y^2-2y$$

$$\Rightarrow x^2+y^2-5x-5y+6=0$$

This is a circle with centre $\left(\frac{5}{2},\frac{5}{2}\right)$ and radius, $r=\sqrt{\frac{13}{4}}=\frac{\sqrt{13}}{2}$.

ANALYSIS

5. (a) $\int_0^2 5x\sqrt{1+x^2}\,dx$ let U = $1+x^2$, $\frac{dy}{dx}=2x$, $dx=\frac{dy}{dx}$.

$$\int 5\times u^{\frac{1}{2}}\frac{du}{dx}=\frac{5}{2}u^{\frac{1}{2}}du=\frac{5}{3}u^{\frac{3}{2}}$$

$$\Rightarrow\int_0^2 5x\sqrt{1+x^2}\,dx=\frac{5}{3}(1+x^2)^{\frac{3}{2}}1_0^2=\frac{5}{3}(5)^{\frac{3}{2}}-\frac{5}{3}$$

$$=\frac{5}{3}(5\sqrt{5-}1)$$

(b) $x\frac{dy}{dx}=1-y^2$, $y(2)=0$

Separating variables

$$\frac{1}{1-y^2}dy=\frac{1}{x}dx \quad , \quad \frac{1}{1-y^2}=\frac{1}{(1-y)(1+y)}=\frac{A}{1-y}+\frac{B}{1+y}$$

$$A(1+y)+B(1-y)=1$$

Put $y=1$, $2A=1\Rightarrow A=\frac{1}{2}$

$y=-1,$ $2B=1\Rightarrow B=\frac{1}{2}$

$$\Rightarrow\frac{1}{2}\int\frac{1}{1-y}dy+\frac{1}{2}\int\frac{1}{1+y}dy=\int\frac{1}{x}dx$$

$$\Rightarrow\frac{1}{2}In(1-y)+\frac{1}{2}In(1+y)=Inx+C,when\quad x=2,y=0\Rightarrow c=-In$$

$$\Rightarrow\frac{1}{2}In\frac{1+y}{1-y}=Inx-In2$$

$$\Rightarrow In\left(\frac{1+y}{1-y}\right)^{\frac{1}{2}}=In\left(\frac{x}{2}\right)$$

$$\left(\frac{1+y}{1-y}\right)^{\frac{1}{2}}=\frac{x}{2}\Rightarrow\frac{y+1}{y-1}=\frac{x^2}{4}$$

$$\Rightarrow 4y+4=yx^2-x^2\Rightarrow(4-x^2)y=-(x^2+4)$$

$$y=\frac{x^2+4}{4-x^2}$$

(c) Let = $\frac{e^{5x}\cos 2x}{In(1-x)}$, $u=e^{5x}\cos 2x$, $v=In(1-x)$

$u^1=5e^{5x}\sin 2x$, $v^1=\frac{1}{1-x}$

$$\frac{dx}{dx}=\frac{\left[e^{5x}In(1-x)(5\cos 2x-2\sin 2x)+e^{5x}\cos 2x\right]}{\left[In(1-x)\right]^2}$$

$$\frac{dy}{dx}=\frac{e^{5x}(1-x)In(1-x)(5\cos 2x-2\sin 2x)+e^{5x}\cos 2x}{In(1-x)^2}$$

6. (a) $y=Ae^{nx}$, $\frac{dy}{dx}=Ane^{nx}$, $\frac{d^2y}{dx^2}=An^2e^{nx}$

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+3y\Rightarrow An^2e^{nx}-4Ane^{nx}+3Ae^{nx}=0$$

$$Ae^{nx}\left[n^2-4n+3\right]=0$$

$n^2-4n+3=0 \quad \Rightarrow \quad (n-3)(n-1)=0 \quad , \quad n=3$ or 1

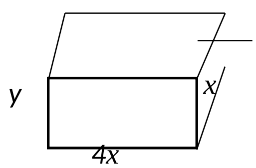
(b) $x=\sin\theta$, $y=1-\cos\theta$ $\frac{dx}{d\theta}=\cos\theta$, $\frac{dy}{d\theta}=\sin\theta$

$$\frac{dy}{dx}=\frac{dy}{d\theta}.\frac{d\theta}{dx}=\frac{\sin\theta}{\cos\theta}=\tan\theta$$

$$\frac{d^2y}{dx^2}=\frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}=\frac{d}{d\theta}\tan\theta\frac{1}{\cos\theta}=\sec^3\theta$$

$$\left(\frac{d^2y}{dx^2}\right)=\sec^6\theta\Rightarrow\left(1+\tan^2\theta\right)^3=\left(1+\left(\frac{dy}{dx}\right)^2\right)^3$$

7. (a)



$$4(4x)+4(x)+4(y)=60$$
$$20x+4y=60$$
$$4y=60-20x$$
$$y=15-5x=5(3-x)$$

(b) $V=L\times W\times h=4x(x)(15$

(c) $\frac{dv}{dx}=120x-60x^2=0,$ $60x(2-x)=0,$ $x=0$, $x=2$

$$\therefore x=2cm$$

Max $V=60(2)^2-20(2)^3=640-160=80\,cm^3$

8. (a) $\int_0^1\frac{X-A}{(X+1)(3X+1)}dx=0$

$$\frac{X-A}{(x+1)(3x+1)}=\frac{P}{x+1}+\frac{Q}{3x+1}$$
$$X-A=P(3x+1)+Q(x+1)$$

Put $x=-1$ $\Rightarrow$ $-1-A=2p \Rightarrow p=\dfrac{1+A}{2}$

$$x=\frac{-1}{3} \Rightarrow -\frac{1}{3}-A=\frac{2}{3}Q \Rightarrow Q=-\left(\frac{1+3A}{2}\right)$$

$$\int_0^1 \frac{1+A}{2(x+1)dx}-\int \frac{1+3A}{2(3x+1)}dx=0$$

$$\left(\frac{1+A}{2}\right)In(x+1)1_0^1-\frac{(1+3A)}{6}In(3x+1)1_0^1=0$$

$$\left(\frac{1+A}{2}\right)In2-\frac{(13A)}{6}In4=0$$

$$\left(\frac{1+A}{2}\right)In2-\frac{(1+3A)}{3}In2=0$$

$$\left[\left(\frac{1}{2}+A\right)-\left(\frac{1+3A}{3}\right)\right]In2=0 \Rightarrow 3+3A-(2+6A)=0$$

$$1=3A \Rightarrow A=\frac{1}{3}$$

(b)
$$\int 2x^2 Inxdx=\frac{2}{3}x^3 Inx-\int \frac{2}{3}x^2 dx=\left[\frac{2}{3}x^3 Inx-\frac{2}{9}x^3\right]_1^8$$

$$=\left(\frac{2}{3}e^3-\frac{2}{9}e^3\right)-\frac{2}{9}$$

$$e^3\left(\frac{2}{3}-\frac{2}{9}\right)-\frac{2}{9}=e^3\left(\frac{4}{9}\right)-\frac{2}{9}$$

9. (a)
$$(x^2-1)\frac{dy}{dx}+2xy=x \Rightarrow \frac{d}{dx}y(x^2-1)=x$$

$$y(x^2-1)=\int xdx$$

$$y=\frac{x^2}{x^2-1}+\frac{x}{x^2-1}$$

(b) Let p be the number of people affected

$$\frac{dp}{dt}\alpha p \Rightarrow \frac{dp}{dt}=kp \qquad \int \frac{dp}{p}=\int kdt$$

$Inp = kt + c$

In 2020 when $t = 0, P = 200$ $\Rightarrow c = In 2000$

$Inp = kt + In 2000 \Rightarrow In\left(\frac{p}{2000}\right) = kt$

$\Rightarrow P = 2000 e^{kt}$

In 2021, when $t = 1, p = 4000$

$\Rightarrow 4000 = 2000 e^{kx1}$ $\Rightarrow 2 = e^{k}$, $k = In 2$

Hence $P = 2000 e^{tIn 21}$

In 2030 when $t = 0$

$p = 2000 e^{10 In 12}$

$\Rightarrow p = 2048000$

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10. (a) $y = In\left(e^{x}\left(\frac{x-2}{x+2}\right)\right)^{\frac{3}{4}}$

$= \frac{3}{4} In\left(e^{x}\left(\frac{x-2}{x+2}\right)\right)$

¿ $\frac{3}{4}\left(Ine^{x} + In(x-2) - In(x+2)\right)$

¿ $\frac{3}{4}x + \frac{3}{4} In(x-2) - \frac{3}{4} In(x+2)$

$\frac{dy}{dx} = \frac{3}{4} + \frac{3}{4} \cdot \frac{1}{x-2} - \frac{3}{4} \cdot \frac{1}{x+2}$

$= \frac{3}{4} + \frac{3}{4(x-2)} - \frac{3}{4(x+2)}$

(b) Given $S = \pi r^{2} + \pi\sqrt{h^{2} + x^{2}}$

$$\frac{s-\pi r^2}{\pi r}=\sqrt{h^2+r^2}$$

$$h^2+r^2=\left(\frac{s-\pi r^2}{\pi r}\right)^2$$

$$h^2=\left(\frac{S-\pi r^2}{\pi r}\right)-r^2$$

$$h^2=\frac{s^2-2s\pi r^2+\pi^2r^4-\pi^2r^4}{\pi^2r^2}$$

$$h^2=\frac{s^2-2\pi sr^2}{\pi 2r^2}$$

Volume, $V=\frac{1}{3}\pi r^2h$

Squaring both sides;

$$V^2=\frac{1}{9}\pi^2r^4h^2 \quad \text{but} \quad h^2=\frac{s^2-2\pi sr^2}{\pi^2r^2}$$

$$9V^2=r^2\left(s^2-2\pi r^2s\right)$$

$$9V\frac{dv}{dr}=2s^2r-8\pi sr^3$$

for max $(V\max)\frac{dv}{dr}=0$

$$0=2s^2r-8\pi r^3 \quad ; \quad 2rs\left(s-4\pi r^2\right)$$

$$r=0 \quad , \quad r^2=\frac{s}{4\pi} \quad \therefore r=\frac{\sqrt{s}}{2\sqrt{\pi}}$$

$$\therefore V\max=\frac{1}{3}\pi\left(\frac{s}{4\pi}\right)\frac{\sqrt{s^2-2\pi s\left(\frac{s}{4\pi}\right)}}{\frac{\pi}{2}\left(\sqrt{\frac{s}{\pi}}\right)}$$

$$=\frac{s}{12}\sqrt{\frac{2s}{\pi}}$$

11. (a) $\int \frac{2e^{2x}}{\sqrt{1-e^{2x}}}=\int \frac{2\left(e^{x}\right)^{2}}{\sqrt{1-\left(e^{x}\right)^{2}}}dx$

Let $u=e^{x}$, $\frac{du}{dx}=e^{x}$

$\therefore \int \frac{2u^{2}}{1-u^{2}}\frac{du}{u}=\int \frac{2u}{\sqrt{1-u^{2}}}du$

Let $p=1-u^{2},$ $\frac{dp}{du}=-2u$, $du=\frac{dp}{-2u}$.

$\int \frac{2u}{vp}.\frac{dp}{2u}=-\int p^{\frac{-1}{2}}dp$

$=-2p^{\frac{-1}{2}}+c$

$¿-2\left(1-u^{2}\right)^{\frac{1}{2}}+c$

$¿-2\left(1-e^{2x}\right)^{\frac{1}{2}}+c$

(b) $\int \frac{12x^{3}+5x}{x^{2}(2x+1)}dx$

$$2x^{3}+x^{2}\overline{)12x^{3}+0x^{2}+5x}$$
$$\underline{-12x^{3}+6x^{2}}$$
$$-6x^{2}+5x$$

$\int \left[6+\frac{5x-6x^{2}}{x^{2}(2x+1)}dx\right]$

$\frac{5x-6x^{2}}{x^{2}(2x+1)}=\frac{A}{x}+\frac{B}{x^{2}}+\frac{c}{2x+1}$

$5x-6x^{2}=Ax(2x+1)+B(2x+1)+cx^{2}$

$x=0, 0=B$

$x=^{-1}\!/_{2}$, $^{-5}\!/_{2}-6\left(^{1}\!/_{4}\right)=c\left(\frac{1}{4}\right)\therefore c=-6$

Coeff. of x $5=A+2B,$ $A=5$

$\int \frac{1^{2}x^{3}+5x}{x^{2}(2x+1)}dx=\int \left(6+^{5}\!/_{x}-\frac{16}{2x+1}\right)dx$

$=6x+5\,Inx-8\,In(2x+1)+c$

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12. (a) (i) Let $y = In\left(\frac{x}{1+x}\right) = Inx - In(1+x)$

$$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{1+x}$$

$$= \frac{(x+1)-x}{x(1+x)} = \frac{1}{x(1+x)}$$

(ii) Let $y = In\left[(x+1)^{\frac{1}{2}} + (x-1)^{\frac{1}{2}}\right]$

$$\frac{dy}{dx} = \frac{1}{(x+1)^{\frac{1}{2}} + (x-1)^{\frac{1}{2}}}\left[\frac{1}{2}(x+1)^{\frac{-1}{2}} + \frac{1}{2}(x-1)^{\frac{-1}{2}}\right]$$

$$= \frac{1}{\left[(x+1)^{\frac{1}{2}} + (x-1)^{\frac{1}{2}}\right]} \cdot \frac{1}{2}\left[\frac{(x-1)^{\frac{1}{2}} + (x+1)^{\frac{1}{2}}}{\left[(x-1)(x+1)^{\frac{1}{2}}\right]}\right]$$

$$= \frac{1}{2(x+1)(x-1)^{\frac{1}{2}}} = \frac{1}{2(x^2-1)^{\frac{1}{2}}}$$

(b) $y = e^{-2x}\cos 4x$

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$$\frac{dy}{dx} = -2e^{-2x}\cos 4x - 4e^{-2x}\sin 4x$$

$$\therefore \frac{dy}{dx} = -2y - 4e^{-2x}\sin 4x$$

$$\frac{d^2y}{dx^2} = \frac{-2dy}{dx} - 4\left[-2e^{-2x}\sin 4x + 4e^{-2x}\cos 4x\right]$$

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 20y = 0$$

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13. $\int \frac{x}{2x^2-x+1}dx$ using splitting numerator method.

$x = A(4x-1) + B$

$\therefore A\frac{1}{4}, B - A = 0$ $\quad A = B = {}^{1}\!/_{4}$

Also $2x^2-x+1=2\left(x^2-\frac{x}{2}+\frac{1}{2}\right)$

$$=2\left(\left(x-\frac{1}{4}\right)^2-\frac{1}{16}+\frac{1}{2}\right)$$

$$¿2\left(x-\frac{1}{4}\right)^2+\frac{7}{8}$$

14. (a) $\int \frac{1}{x^3\sqrt{(x^2-4)}dx}$ let $x=2\sec\theta$

$dx=2\sec\theta\tan\theta d\theta$

$$\int \frac{1}{(2\sec\theta)^3}\cdot\frac{1}{(\sqrt{2\sec\theta})^2_{-4}}.2\sec\theta\tan\theta d\theta$$

$$\int \frac{1}{(2\sec\theta)^2}\cdot\frac{\tan\theta d\theta}{2\tan\theta}$$

But $\sec\theta=\frac{x}{2}$

$$\int \frac{1}{8}\cos^2\theta d\theta$$

$\cos\theta=\frac{2}{x}$

$$\frac{1}{8}\int\frac{1}{2}(1+\cos 2\theta)d\theta$$

$\sin\theta=(1-\cos^2\theta)^{\frac{1}{2}}$

$$\frac{1}{16}\left[\theta+\frac{1}{2}\sin 2\theta\right]$$

$=\left(1-\frac{4}{x^2}\right)^{\frac{1}{2}}$

$=\frac{(x^2-4)}{x}$

$$\frac{1}{16}\left[\sec^{-1}\left(\frac{x}{2}\right)+\frac{1}{2}2\cos\theta\sin\theta\right]$$

$$\frac{1}{16}\left[\sec^{-1}\frac{x}{2}+\frac{1}{2}.\frac{2}{x}.2\frac{x^2-4}{x}\right]+c$$

$$\frac{1}{16}\sec^{-1}\frac{x}{2}+\frac{\sqrt{(x^2-4)}}{8x^2}+c$$

(b) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x^2\sin 2(x^3)dx$ let $u=x^3$

$du=3x^2du$

$$\int x^2 . sn^2 u \frac{du}{3x^2}$$

$$1\frac{1}{3}\int \sin 2\, du = \frac{1}{3}\int \frac{1}{2}(1-\cos 2u)\, du$$

$$= \frac{1}{6}\left[u - \frac{1}{2}\sin 2u\right]$$

$$= \frac{1}{6}\left[x^3 - \frac{1}{2}\sin\left(x^3\right)\right]^{\frac{\pi}{2}}$$

$$¿ \frac{1}{6}\left(\left(\frac{\pi}{3}\right)^3 - \frac{1}{2}\sin 2\left(\frac{\pi}{2}\right)^3\right) - \frac{1}{6}\left(\frac{\pi}{4}\right)^3 - \frac{1}{2}$$

$$¿ \frac{\pi^3}{48} - \frac{1}{12}(0) - \frac{\pi^3}{1296} - 1 = \frac{26\pi^{-3}}{1296} - 1$$

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15. (a) $$y = \frac{e^{\cos 2x}}{x^x \cos 2y}$$

$$Iny = Ine^{\cos 2x} - Inx^x - In\cos y$$

$$\frac{1}{y} = \cos x$$

$$Iny\cos 2x - xIhx - In\cos y$$

$$\frac{1}{y} = \frac{dy}{dx} = -2\sin 2x - \left[x . \frac{1}{x} + Inx\right] - \frac{1}{\cos^3 y}$$

$$\frac{1}{y}\frac{dy}{dx} = -2\sin 2x - 1 - In + 2\frac{\sin 2y}{\cos 2y} . \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx}\left(\frac{1}{y} - 2\tan 2y\right) = 2\sin 2x - 1 - Inx$$

$$\therefore \frac{dy}{dx} = \frac{y(-2\sin 2x - 1 - 1\,Inx)}{1 - 2y\tan 2y}$$

$$= \frac{y(2\sin 2x + 1 + Inx)}{2y\tan 2y - 1}$$

(b) $y=\theta-\cos\theta$, $\dfrac{dy}{d\theta}=1+\sin\theta$

$x=\sin\theta$, $\dfrac{dx}{d\theta}=\cos\theta$

$$\frac{dy}{dx}=\frac{dy}{d\theta}.\frac{d\theta}{dx}=1+\sin\theta.\frac{1}{\cos\theta}=\frac{1+\sin\theta}{\cos\theta}$$

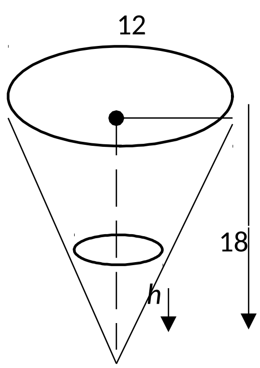
$$\frac{d^2y}{dx^2}=\frac{d}{d\theta}\left(\frac{dy}{d\theta}\right).\frac{d\theta}{dx}\,;u=1+\sin\theta$$

$$\frac{dy}{dx}=\cos\theta \quad , \quad \frac{dv}{d\theta}=-\sin\theta$$

$$\frac{d^2y}{dx^2}=\left(\frac{\cos\theta(\cos\theta)-(1+\sin\theta)(-\sin\theta)}{\cos^2\theta}\right).\frac{1}{\cos\theta}$$

$$¿\frac{\cos^3\theta+\sin\theta+\sin^2\theta}{\cos^3\theta}=\frac{1+\sin\theta}{\cos^3\theta}$$

(b)



$$\frac{18}{h}=\frac{12}{r},r=\frac{12}{18}h=2/3\,h$$

$$\therefore V=\frac{1}{3}\pi r^2h=\frac{1}{3}\pi\left(\frac{2}{3}h\right)^2h$$

$$¿\frac{4\pi h^3}{27}$$

$$\frac{dv}{dh}=\frac{4}{9}\pi h^2$$

$$\therefore\frac{dh}{dt}=\frac{dv}{dh}.\frac{dh}{dv}$$

$$\frac{2..9}{4\pi h^2}=\frac{18}{4\pi h^2}$$

$$\frac{dh}{dt}=\frac{9}{2\pi(6^2)}=\frac{9}{72\pi}=\frac{1}{8\pi}\,cm/s$$

16. (a) $y=\cos\left(Inx\right)$ Let $u=Inx$

$\frac{du}{dx}=\frac{1}{x}$

$$\frac{dy}{dx}=\frac{dy}{dh}.\frac{du}{dv}$$

$y=\cos u.$ $\frac{dy}{du}=-\sin u$

$$=-\sin u.\frac{1}{x} \qquad \frac{dy}{dx}=x\frac{dy}{dx}$$

$$\frac{dy}{dx}=-\frac{1}{x}\sin\left(Inx\right).=-x^{-1}\sin\left(Inx\right)$$

$$\frac{d^2y}{dx^2}=-x^{-1}\frac{\cos\left(Inx\right)}{x}+x^{-2}\sin\left(Inx\right)$$

$$\frac{d^2y}{dx^2}=-x-2\cos\left(Inx\right)+x^{-2}\sin\left(Inx\right)$$

$$=\frac{\cos\left(Inx\right)}{x^2}+\frac{\sin\left(Inx\right)}{x^2}$$

$$x^2\frac{d^2y}{dx^2}=-y+\left(\frac{dy}{dx}\right)$$

$$\therefore x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$$

(b) $y=\frac{e^{2x}-1}{e^{2x}+1}$, $y\left(e^{2x}+1\right)=e^{2x}-1$

$$ye^{2x}-e^{2x}=-1-y$$

$$e^{2x}\left(y-1\right)=-\left(1+y\right)$$

$$e^{2x}=\frac{1+y}{1-y}$$

Take $\log_e$ both sides

$$Ine^{2x}=In\left(\frac{1+y}{1-y}\right)$$

$$2x = In\left(\frac{1+y}{1-y}\right)$$

$$x = \frac{1}{2} In\left(\frac{1+y}{1-y}\right)$$

(c) (i) $e^{Inx} = x$

Let $y = e^{Inx}$, take $\log_e$ both sides

$$Iny = Ine^{Inx}$$

$$Iny = InxIne \quad . \qquad \therefore e^{Inx} = x$$

$$Iny = Inx \qquad \therefore y = x$$

(ii) $e^{2Inx} = e^{Inx-2}$

Take In both sides

$$Iny = Inx^{-2} Ine$$

$$Iny = Inx^{-2}$$

$$\therefore = yx^{-2} = \frac{1}{x^2}$$

$$\therefore e^{-2Inx} = \frac{1}{x^2}$$

17. Show that $\dfrac{\sin 3A}{1+2\cos 2A} = SinA$

$$\frac{3\sin A - 4\sin 3A}{1+2\cos 2A}$$

$$\sin A \frac{(3-4\sin 2A)}{1+2\cos 2A}$$, but $\cos 2A = 1-2\sin A$, $-2\sin^2 A = \cos 2A - 1$

$$\sin A\left(\frac{1+2\cos 2A}{1+2\cos 2A}\right) = \sin A$$

$\frac{\sin 45}{1+2\cos 30}$, $\quad 3A=45$, $\quad A=15$

$2A=30$

$$\frac{\sin 45}{1+2\cos 30}=\frac{\frac{\sqrt{2}}{2}}{1+2.\frac{\sqrt{3}}{2}}$$

$$=\frac{\sqrt{2}}{2}.\frac{(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}$$

$$=\frac{\sqrt{2}}{2}.\frac{1-\sqrt{3}}{1-3}=\frac{(\sqrt{2})-(1-\sqrt{3})}{2}=\frac{\sqrt{3}-1}{2\sqrt{2}}$$

(b) $8\cos 3\theta\cos 2\theta\cos\theta-1=\frac{\sin 7\theta}{\sin\theta}$

18. (a) $\sin(2\sin^{-1}x+\cos^{-1}x)$

Let $A=2\sin^{-1}x$, $\sin A/2=x$, $\cos A/2=(1-x^2)^{\frac{1}{2}}$

$B=\cos^{-1}x$ $\quad \cos B=x$, $\sin B=\sqrt{1}-x^2$

$$\sin(A+B)=\sin A\cos B+\cos A\sin B$$

$$=2\sin\frac{A}{2}\cos A/2\cos B+\left(1-2\sin^2 A/2\right)\sin B$$

$$=2x(1-x^2)^{\frac{1}{2}}x+(1-2x^2)(1-x^2)^{\frac{1}{2}}$$

$$=2x^2(1-x^2)^{\frac{1}{2}}+(1-2x^2)(1-x^2)^{\frac{1}{2}}$$

$$=(1-x^2)^{\frac{1}{2}}(2x^2+1-2x^2)$$

$$=(1-x^2)^{\frac{1}{2}}$$

$\sin(A+B)=(1-x^2)^{\frac{1}{2}}$ as required.

(b) $10\sin+\cos x+12\cos 2x$ in form $\sin(2x+\alpha)$

$5.2\sin x\cos x+12\cos 2x$

Let $5\sin 2x+12\cos x=R\sin(2x+\alpha)$

$5\sin 2x+12\cos x=R\sin 2x\cos\alpha+R\cos 2x\sin\alpha$

$\therefore R\cos\alpha=5$, (i) $R\sin\alpha=12$, (ii) $\therefore \tan\alpha={}^{12}\!/\!_{5}$

Squaring and add (i) to (ii)

$R^2(\cos^2\alpha+\sin^2\alpha)=5^2+12^2=169$
$R=13$
$\therefore 5\sin 2x+12\cos 2x=13\sin(2x+\alpha)$, Where $\tan^{-1}\left({}^{12}\!/\!_{5}\right)$

Squaring and add (i) to (ii)

$R_2(Cos^2\alpha+\sin^2_\alpha)=5^2+12^2=169$

$R=13$

$\therefore 5\sin 2x+12\cos 2x=13\sin(2x+\alpha)$, where $\tan^{-1}\left({}^{12}\!/\!_{5}\right)$

$\therefore 10\sin x\cos x+12\cos 2x+7=13\sin(2x+\alpha)+7$

$\therefore 13\sin(2x+\alpha)+7=0$

$\sin(2x+\alpha)={}^{-7}\!/\!_{13}=-0.585$

$2x+\alpha=\sin-1\left({}^{7}\!/\!_{13}\right).=-35.58,$ 237.42

$2x+67.38=-35.58$, 237.42

$2x=-102.96$, 170.04

$x-57.48$, 85.02

(c) $$\frac{\sin 8\theta\cos\theta-\sin 6\theta\cos 3\theta}{\cos 2\theta\cos\theta-\sin 3\theta\sin 4\theta}=$$

Splitting using factor formula

$A+B=16\theta$
$+A-B=2\theta$
$2A=18\theta$, $A=9\theta$

Subst. $2B=14\theta$, $B=7\theta$

$A+Q=12\theta(i)$
$P-Q=6\theta(ii)$
$(i)-(ii)D=9\theta$
$(i)-(ii)Q=3\theta$

Then

$A+B=4\theta (i)$
$A-B=2\theta (ii)$

(i) + (ii) $\quad A=3\theta$

(i) ¿(iii) $\quad B=\theta$

$P+\theta=6\theta-(i)$
$P-Q=8\theta(i)$
$(i)+(ii)P=7\theta$
$(i)-(ii)Q=-\theta$

$$\therefore \frac{\frac{1}{2}(\sin 9\theta+\sin 7\theta)-\frac{1}{2}(\sin 9\theta+\sin 3\theta)}{\frac{1}{2}(\cos 3\theta+\cos\theta)+\frac{1}{2}(\cos 7\theta-\cos(-\theta))}$$

$$\frac{\frac{1}{2}\sin 7\theta-\frac{1}{2}\sin 3\theta}{\frac{1}{2}\cos 3\theta-\frac{1}{2}\cos 7\theta}$$

$$=\frac{2\cos 5\theta\sin 2\theta}{-2\sin 5\theta\sin(-2\theta)}=\tan 2\theta-$$

19. (a) $a=\sin\theta+\sin\beta$

$$a=2\sin\left(\frac{\theta+\beta}{2}\right)\cos\left(\frac{\theta-\beta}{2}\right)$$

$$a^2=4\sin^2\left(\frac{\theta+\beta}{2}\right)\cos^2\left(\frac{\theta-\theta}{2}\right)$$

$$b=\cos\theta+\cos\beta$$

$$b=2\cos\left(\frac{\theta+\beta}{2}\right)\cos\left(\frac{\theta-\beta}{2}\right)$$

$$b^2=4\cos^2\left(\frac{\theta+\beta}{2}\right)\cos^2\left(\frac{\theta-\beta}{2}\right)$$

$$\frac{1}{4}(a^2+b^2)=\frac{1}{4}\left(4\sin^2\left(\frac{\theta+\beta}{2}\right)\cos^2\frac{(\theta+\beta)}{2}+4\cos^2\frac{(\theta+\beta)}{2}\right)\cos^2\frac{(\theta-\beta)}{2}$$

$$=\cos^2\left(\frac{\theta-\beta}{2}\right)\left[\sin^2\left(\frac{\theta+\beta}{2}\right)+\cos^2\left(\frac{\theta+\beta}{2}\right)\right]$$

$$¿\cos^2\left(\frac{\theta-\beta}{2}\right).1$$

(b) Let $8\cos x+6\sin x=R\cos(x-\alpha)$

$$8\cos x+6\sin x=R\cos x\cos\alpha+R\sin\times\sin\alpha$$

$$\therefore R\cos\alpha=8-(i)$$

$$R\sin\alpha=6-(ii)$$

Eliminating α $R^2=8^2+6^2=100, R=10$

“ $R,$ $\tan\alpha=\frac{6}{8},$ $\alpha=\tan^{-1}\left(\frac{3}{4}\right)=36.87^0$

$$\therefore 8\cos x+6\sin x=10\cos(x-36.87^0)$$

$$y=\frac{1}{10\cos(x-36.87)^0+15} \qquad \cos(x-36.8)=-1$$

$$y_{\max}=\frac{1}{-10+15}=\frac{1}{5} \quad , \quad x=36.87+180=216.87$$

$$y_{\min}=\frac{1}{10+15}=\frac{1}{25} \quad , \quad \cos(x-36.87)=1$$

$$x=36.87$$

$\therefore$ main value $\left(\frac{1}{5}, 216.87\right),$

Min value $\left(\frac{1}{25}, 36.87\right)$

20. (a) Line $2x-y+3=0$ $c(-4,5)$

Radius = distance from $(-4,5)$ to line $2x-y+3=0$

$$R=\left|\frac{2(-4)(5)+3}{\sqrt{2^2+(-1)^2}}\right|=\left|\frac{-10}{\sqrt{5}}\right|=2\sqrt{5}$$

For $x^2+y^2-2ax-2ax-2by+c=0$

$c=a^2+b^2-r^2$

$¿(-4)^2+(5)^2-(2\sqrt{5})$

$¿16+25-20=21$

$\therefore eqn$ of circle $x^2+y^2+8x-10y+21=0$

(b) $x^2+y^2-6x-12y+40=0$ & $x^2+y^2+4y-16=0$

$C_1(3,6)$ $R_1=3^2+6^2-40=5$ $C_2(0,2)\,R_2=6^2+2^2-(-16)=20$

For $\overline{C_1C_2}^2=(3-0)^2+(6-2)^2$

$=9+16=25$

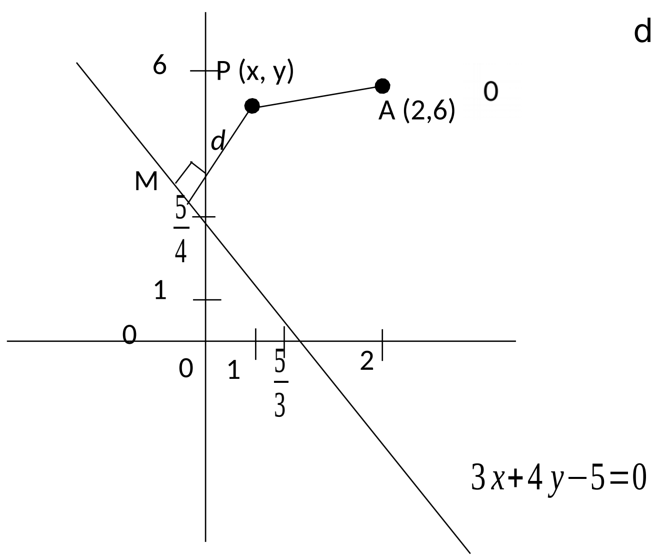
For Orthogonal circle:

$R_{1^2}+R_{2^2}=\overline{C_1C_2}^2$

$5+20=25$

So they are orthogonal

(c)



d from

$$d=|\frac{av_1+by_1+c}{\sqrt{a^2+6^2}}|$$

$\overline{PA}^2=\overline{PM}^2$

$(x-2)^2+(y-6)^2=x^2+y^2-4x-12y+4+36$

$PM=R(v,y)\,from\,3x+4y-5=0$

$$d=|\frac{3x+4y-5}{\sqrt{3^2+4^2}}|=\frac{3x+4y-5}{5}$$

$$\overline{PA}^2 = \overline{PM}^2$$

$$v^2 + y^2 - 4x - 12y + 40 = \left(\frac{3x + 4y - 5}{5}\right)^2$$

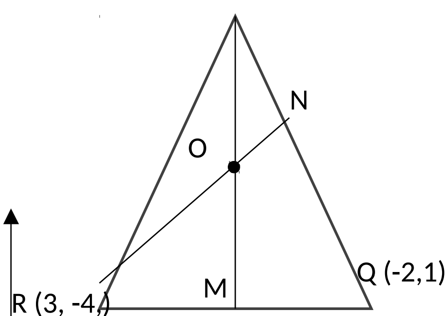
$$25\left(x^2 + y^2 - 4x - 12y - 40\right) = (3x)^2 + 2(3x)(4y - 5) + (y - 5)^2$$

$$25\left(x^2 + y^2 - 4x - 12y + 40\right) = 9x^2 + 24xy - 30x + 16y^2 - 40y + 25$$

$$16x^2 + 9y^2 - 70x - 260y - 24xy + 975 = 0$$

21. N

22.



RN and PM are altitudes

Grade $PQ = \frac{-1-1}{-6-{}^{-}2} = \frac{-2}{-4} = \frac{1}{2}$ so gradient $RN = -2$

Eqn. of $RN \Rightarrow \frac{y+4}{x-3} = -2$, $y + 2x + 4 - 6 = 0$, $y + 2x - 2 = 0 = (i)$

Grad of $RQ = \frac{1+4}{-2-3} = \frac{5}{-5} = -1,$ grad $PM = 1$

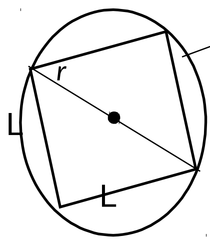
eqn of $PN, \frac{y+1}{x+6} = 1$ $\therefore = y = x + 5 - (ii)$

Solving simultaneously (1) & (2)

$y+2-2=0$
$y=x+5$
$x+5+2x-2=0$
$3x=-3, x=-1$
$y=1+5=4$

Ortho centre $(-1,4)$

(b)



$x^2+y^2-4x-3y-36=0$

Centre $\left(2,\frac{3}{2}\right)$

Radius, r.

$$r^2=2^2+\left(\frac{3}{2}\right)^2-(-36)$$

$$\therefore r=\frac{13}{2}$$

Diagonal $=2r=13\,units$

Therefore Area of square = $L\times L$

But $2L^2=13^2$ $\quad l^2=\frac{169}{2}=\frac{13}{\sqrt{2}}$

$$A=\frac{13}{\sqrt{2}}.\frac{13}{2}=\frac{169}{2}=84.5\,squnits$$

(c) $2x-3y-4=0$, $y=\frac{2}{3}x-4.$ $\quad m=\frac{2}{3}$

Using $\theta=tain-1\left(\frac{m_1+m_2}{1-m_1m_2}\right), \tan\theta=\frac{m_1+m_2}{1-m_2m_2}$

$$2=\frac{\frac{2}{3}+m^2}{1-\frac{2}{3}m^2}$$, $m_2=\frac{4}{7}$

$eqn.\ \dfrac{y-^{-}2}{x-3}=4\!\!\diagup\!\!7$ $\qquad \therefore 7(y+2)=4(x-3)$

$7y-4x+26=0$

---

23. (a) $A+P(x,y)$, $\dfrac{dy}{dx}=\dfrac{2-x}{y-3}$

$\therefore (y-2)dy=(2-x)dx$

Integrating $\dfrac{1}{2}y^2-3y=2x-\dfrac{1}{2}x^2+B$

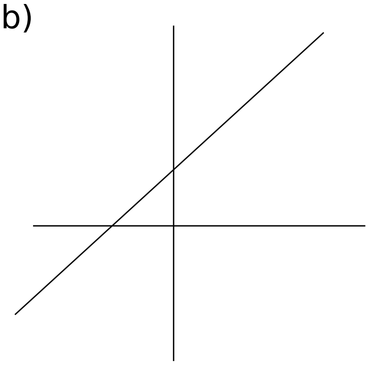
$\dfrac{1}{2}(y^2+x^2)-2x-3y+B=0$

Hence $x^2+y^2-4x-6y+C=0$

But centre $C=2^2+3^2-4^2=-3$

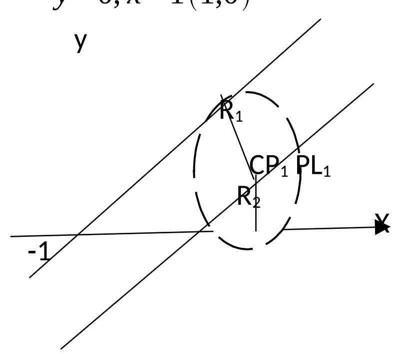
$\therefore x^2+y^2-4x-6y-3=0$

(b)



$x-y-1=0, x=0, y=-1\ (0,-1)$

$y=0, x=1\ (1,0)$



For $4x-3y+4=0$

$x=0, y=4\!\!\diagup\!\!3, \left(0, 4\!\!\diagup\!\!3\right)$

$y=0 \qquad x=4$

For $4x-3y+4=0 \qquad (P, P-1)$

$$R1=\left|\frac{4P-3(p-1)+4}{\sqrt{4^2+(-3)^2}}\right|=\frac{4p-3p+3+4}{5}=\frac{p+7}{5}-(i)$$

$R_2=p-$

$But R_2-R_1$

Then $\dfrac{p+7}{5}=p-1$

$p+7=5p-5$

$12=4p$, p = 3

Centre $(3,2)$ and $R=p-1=2$

For $x^2+y^2-2ax-2by+c=0$

$c=a^2+b^2-r^2$

¿ $3^2+2^2-2^2=9$

∴ Equation of circle: $x^2+y^2-6x-4y+9=0$

---

24. (a) $a=\begin{pmatrix}3\\-2\end{pmatrix} b=\begin{pmatrix}1\\-3\\5\end{pmatrix} c=\begin{pmatrix}2\\1\\-4\end{pmatrix}$

$$a.c=\begin{pmatrix}3\\-2\\1\end{pmatrix}.\begin{pmatrix}2\\1\\-4\end{pmatrix}=6-2-4=0$$ ∴ a

is perpendicular to c and also $a+\lambda c$ or a¿$k\frac{1}{2}\Longrightarrow A$ triangle.

Hence area

$|a|=$¿ $\sqrt{3^2+(-2)^2+(1)2}=\sqrt{14}$

$|c|$ $=\sqrt{2^2+1^2+(4)^2}=\sqrt{14}$

Area = $\frac{1}{2}\times\sqrt{14}\times\sqrt{21}=\frac{1}{2}.\sqrt{2}.\sqrt{7}.\sqrt{3}.\sqrt{7}$

$$= \frac{7}{2}\sqrt{6} \text{ sq. units}$$

(b) $2x+3y-z=4(i)$ & $x-y+2z=5-(2)$

Eliminate z

$$\left| \begin{array}{l} 2x+3y-z=4-(i) \\ x-y+2z=5-(ii) \end{array} \right. \bar{I}$$

$(i)+(ii)$

$$5x+5y=13$$
$$5x=13-5y$$
$$x=\frac{13-5y}{5}$$

Eliminate y

$(i)+3(2)$

$$2x+3y-z=4$$
$$3x-3y+6z=15$$
$$\therefore 5x+5z=19$$
$$x=\frac{19-5z}{5}$$

$$x=\frac{y-\frac{13}{5}}{-1}$$

and

$$x=\frac{z-\frac{19}{5}}{-1}$$

Cartesian eqn.

$$x=\frac{y-\frac{13}{5}}{-1}=\frac{z-\frac{19}{5}}{-1}$$

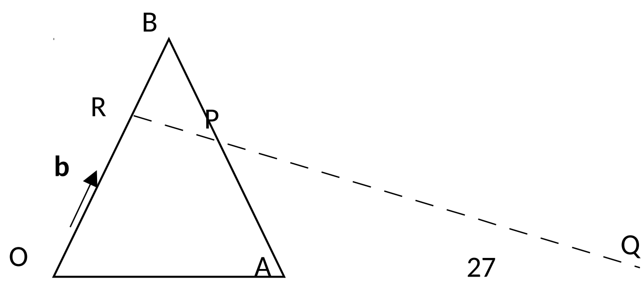
Or

$$=\begin{pmatrix} 0 \\ \frac{13}{5} \\ \frac{19}{5} \end{pmatrix}+\lambda\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$

The vector equation of the line

$$r=¿ \begin{pmatrix} 0 \\ \frac{13}{5} \\ \frac{19}{5} \end{pmatrix}+\lambda\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$

(c)



$\longrightarrow \boldsymbol{a}$ – – – – – – – – – – – – –

$\frac{\overline{QR}}{RB}=\frac{4}{1}$ $\frac{\overrightarrow{BP}}{\overrightarrow{PA}}=\frac{2}{3}, \overrightarrow{BA}=a-b$

$¿=\frac{4}{5}b$; $\overrightarrow{BP}=\frac{2}{5}BA=\frac{2}{5}(a-b)$

$\therefore$ $RB=\frac{1}{5}b$ $PA=\frac{3}{5}BA=\frac{3}{5}(a-b)=\frac{2}{5}a+\frac{3}{5}b$

$¿\frac{1}{5}(2a+3b)$

$OQ=k\,OA=¿+\mu RP\ but\ RP=RB+BP$

$\therefore ka=\frac{4}{5}b+\mu\left(\frac{1}{5}b+\frac{2}{5}a-\frac{2}{5}b\right)$

Comparing both sides

$\frac{4}{5}-\frac{1}{5}\mu=0$

$\mu=4$

$\therefore u=\frac{1}{5}.2\mu=\frac{2\times 4}{5}=\frac{8}{5}$

$\therefore OQ=\frac{8}{5}a$

25. (a) $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{5}=\mu$ i.e. $x=5+4\mu$, $y=7+4\mu$,

$z=-3+5$

$\frac{x-8}{7}=\frac{y-4}{7}=\frac{z+5}{3}=\lambda$ i.e. $x=8+7\lambda$, $y=4+\lambda$, $z=-5+3\lambda$

At point intersection

$5+4\mu=8+7\lambda$; $\therefore 4\mu-7\lambda=3-(i)$

$7+4\mu=4+\lambda$; $4\mu-\lambda=-3=(ii)$

$-3+5\mu=-5+3\lambda$; $\therefore 5\mu-3\lambda=-8-(iii)$

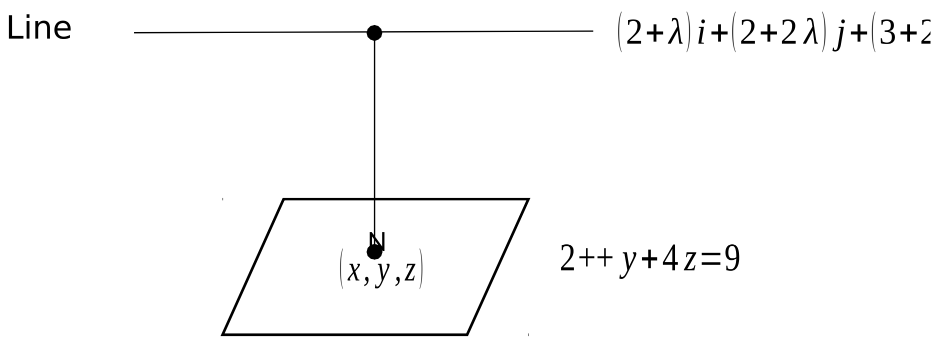
$\therefore (i)-(ii)-6\lambda=46$

$\lambda=-1$

$\therefore x=8-7=1$, $y=4-1=3,$ $z=-5-3=-8$

$(1,3,-8)$

(b) $\frac{x-2}{1}=\frac{y-2}{2}=\frac{z-3}{2}$, $2x+y+4z=9$



$PN;$ $x=2+\lambda$, $y=2+2\lambda$, $z=3+2\lambda$

At $x=3,$ $2+\lambda=3$, $\lambda=1$

So $P(3,4,5)$

$\therefore$ $NP=¿$ normal to plane $2x+y+4z=9$

$OP-ON=\begin{pmatrix}2\\1\\4\end{pmatrix}$ $\therefore$ $x=1, y=3, z=1$

$\therefore N(1,3,1)$